MATH 2415 Homework 12 - Due Nov.15

1. (2 pts) Let A and B be the following matrices:

$$A = \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix}, \text{ and } B = \begin{pmatrix} -2 & 3 \\ -5 & 1 \end{pmatrix}.$$

(a). Compute $A + B$.
(b). Compute AB and BA, and show they are not equal.

2. (3 pts) Compute the following matrix multiplcation,

$$\begin{pmatrix} 1 & -1 & 3 \\ 2 & 5 & -3 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 1 & 0 \\ -1 & 3 \end{pmatrix}.$$

What happens if we try to multiply the matrices in the reverse order?

3. (2 pts) Let

$$A(t) = \begin{pmatrix} \sin(t) & \cos(t) & 0 \\ e^t & e^{-t} & 1 \end{pmatrix}.$$

(a). Compute $A'(t)$.
(b). Compute $\int_0^1 A(t)dt$.

4. (4 pts) Compute the determinants of the following matrices.
(a).

$$\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}.$$

(a).

$$\begin{pmatrix} 2 & 3 & 1 \\ 5 & 7 & -1 \\ 2 & 1 & 0 \end{pmatrix}.$$

5. (2 pts) Which of the following is the inverse of

$$\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}?$$

I. $\begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}$ II. $\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$ III. $\begin{pmatrix} 3 & -2 \\ -2 & 1 \end{pmatrix}$

Explain your result.

6. (4 pts) Which of the following is the inverse of

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 2 & 3 \end{pmatrix}?$$

I. $\begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 2/3 & -2/3 & 1 \end{pmatrix}$ II. $\begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 2/3 & -2/3 & 1 \end{pmatrix}$ III. $\begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 2/3 & -2/3 & 1 \end{pmatrix}$

Explain your result.

7. (4 pts) Show that the following set of vectors is linearly independent (using the definition of linearly independent set):

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \right\}.$$

8. (4 pts) Show that the following set of function-valued vectors is linearly dependent (using the definition of linearly independent set):

$$\left\{ \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} \sin^2(t) \\ \cos^2(t) \\ 1 \end{pmatrix}, \begin{pmatrix} \cos^2(t) \\ \sin^2(t) \\ 1 \end{pmatrix} \right\}.$$

9. (6 pts) Compute the eigenvectors and eigenvalues of

$$\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}.$$

10. (9 pts) Consider the matrix

$$A = \begin{pmatrix} -1 & 2 & 1 \\ 2 & 2 & 2 \\ -1 & -2 & -3 \end{pmatrix}$$

and the vectors

$$\overrightarrow{v_1} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \overrightarrow{v_2} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \overrightarrow{v_3} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

(a). Show that $\overrightarrow{v_1}, \overrightarrow{v_2}$, and $\overrightarrow{v_3}$ are eigenvectors with eigenvalues $2, -2$, and -2 , respectively, by computing $A\overrightarrow{v_1}, A\overrightarrow{v_2}$ and $A\overrightarrow{v_3}$.

(b). Write down the matrix $A - \lambda I$, where λ is a variable.

(c). Compute $\det(A - \lambda I)$.

(d). In (c) you get that $\det(A - \lambda I)$ is a polynomial of λ, and we denote this polynomial as $P(\lambda)$. As a polynomial, what is the degree of $P(\lambda)$? What are the roots of $P(\lambda)$?